A semi-empirical unified model of strain fatigue life for insulation plastics

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Received: 13 July 2009 / Accepted: 30 September 2009 / Published online: 9 October 2009 Springer Science+Business Media, LLC 2009

Abstract The paper focuses on modeling the strain fatigue lives of three commonly used cable insulation polymers, namely (1) polyvinyl chloride, (2) crosslinked polyethylene, and (3) polyphenylene ether under selected strain and temperature ranges. On the basis of results obtained from their fatigue tests, Coffin–Manson model, mean/maximum strain fatigue model, and a set of new semi-empirical equations were applied to establish the relationship between fatigue lives and strains. The unified strain model, herein we name it the Wei–Wong model, is developed to predict the fatigue lives of three polymers studied and their prediction capability was examined using our experimental data. It was found that the proposed Wei– Wong model can provide a better life prediction compared to the experimental data and other methods in the literature at selected temperatures, namely -40 , 25, and 65 °C.

Introduction

Polyvinyl chloride (PVC), crosslinked polyethylene (XLPE), and polyphenylene ether (PPE) are the principal insulation materials used in the wire harnessing industry [[1,](#page-7-0) [2](#page-7-0)]. Due to their high mechanical properties, excellent electrical and thermal resistance, durability, and low density $[3-6]$, they constitute the main components for insulation and jacketing applications in automotive and aeronautical wiring harnesses. Of critical importance is a systematic evaluation of the strain fatigue lives of insulation plastics prevalent in electro-mechanical and life-

threatening components. The ability to predict the strain fatigue life of polymers using a semi-empirical approach will be valuable for the manufacturing industry, which is increasingly focused on light-weight, green, and energyefficient developments.

At present, there are very sparse experimental data on strain fatigue life available for the wire harnessing and insulation plastics industry. Most research on fatigue of polymers is focused on load-controlled conditions, which do not apply sufficiently in cabling and wiring industry. Low-cycle fatigue models for alloys and metals are well studied and widely applied to fatigue life prediction [[7,](#page-7-0) [8](#page-7-0)]. Tomkins and Biggs [\[9](#page-7-0)] verified Coffin–Manson model in nylon and obtained a fatigue exponent of 0.23. Nylon shows the continuous nonlinear softening and obeys a plastic strain criterion of fatigue life, as expressed by the Coffin–Manson relationship. Recently, the fatigue properties of an industrial glass–fiber reinforced composite were reported by Rosenkranz et al. [[10\]](#page-7-0) under load or strain control at 77 K and 10 Hz. The results show that the sample width does not affect the fatigue behavior at both room temperature and 77 K. While the sample thickness exceeds the recommended thickness, 3 mm, it leads to an increase of the fatigue lifetime. Kultural and Eryurek [[11\]](#page-7-0) investigated the fatigue behavior of polypropylenes filled with three different loadings of calcium carbonate to verify the effects of filler content on fatigue performance. It was found that increasing the filler content reduces the fatigue performance of PP. Under various ambient temperatures, Takahara et al. [\[12](#page-7-0)] explored the variations of dynamic viscoelastic properties using cyclic straining to clarify fatigue mechanisms and fatigue criteria of plasticized PVC. The failure occurs when the effective energy loss reaches a certain magnitude depending on an ambient temperature and the fatigue criterion was represented schematically from a standpoint of

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self-heating. Considering the nonlinear effect of stress ratio and load frequency on the fatigue life, Epaarachchi and Clausen [[13](#page-7-0)] predicted the fatigue behavior for glass fiberreinforced plastic composites using a well-defined minimum number of tests. The semi-empirical model is based on constant amplitude and constant frequency cyclic loading. To date, very little has been done on modeling strain fatigue lives of insulation plastics and the capability to predict such strain fatigue lives is critically needed [\[14](#page-7-0)]. The Wei–Wong model presented herein will provide a useful platform for future modeling work.

In this model, the effects of total strain amplitude, maximum, minimum, and mean strains will be unified at three different selected temperatures. This model is based on the assumption that we apply cyclic loadings equivalent to tension–tension mode on the studied specimens. The predicted fatigue lives from the new model were compared to the experimental data we obtained under the given conditions. The unified strain model was found to have a closer agreement with experimental results on fatigue lives in comparison to the single strain models for PVC, XLPE, and PPE.

Experiment results

The studied polymers, PVC, XLPE, and PPE in the form of pellets and their sources, processing conditions and tested results are reported in [[15\]](#page-7-0). The setup for the uniaxial tensile tests is shown in Fig. 1a using an Instron 5582 at a quasistatic rate with a load cell of 10 kN in room temperature. The tensile tests were conducted separately to examine tensile strengths and breaking strains. The experimental data are summarized in Table 1. The setup for strain fatigue tests is shown in Fig. 1b and c using a fully automated, closed loop, servo-hydraulic mechanical testing machine

Fig. 1 The test chamber with an assembled grip setup for fatigue tests. a Two extensometers mounted on the center of specimen to measure strains at the transverse and longitudinal directions. b The experimental setup with sample mounted between the grips before testing. c The specimen after pre-straining but before applying the cyclic displacement

Table 1 Average tensile strength and breaking strain values for PVC, XLPE, and PPE at a loading rate of 20 mm/min

Material	Tensile strength (MPa)	Breaking strain $(\%)$		
PVC	12.9	230		
XLPE	10.9	224		
PPE	43.8	62		

(Instron model 8500) at a frequency of 5 Hz for PVC and XLPE, and 2 Hz for PPE, for practical reasons to enable observations of specimens breaking at failure.

The purpose of the displacement controlled fatigue tests is to produce fruitful information and insights in the influence of strain amplitude on fatigue life of insulation plastics. At an infinitesimally small level of applied strain amplitude, the polymers can be cyclically loaded indefinitely. Due to the limitations imposed by test time, an arbitrary cutoff value of $10⁶$ cycles was used to establish the endurance limit. The tests at cold temperature are limited to around 25,000 cycles due to a finite supply of liquid nitrogen for the cooling tests.

At the three different temperatures: -40 , 25, and 65 °C, the maximum, minimum, and mean strain values versus the number of cycles to failure for three materials are obtained in fatigue testing and listed in Table [2](#page-2-0) [[15\]](#page-7-0). It is noted that at a lower temperature the specimen exhibits lower strain fatigue lives; while at a higher temperature the specimen exhibits much higher strain fatigue lives compared to the room temperature in case of PVC and XLPE. This is because when temperature decreases, most polymers go through glass transitions and they become embrittled due to restricted flows; whereas at higher temperatures polymer chains are able to slither over one another and the material is more ready to flow. For PPE, it is quite the reverse for the relationship between strain and temperature. The

Materials	Temperature (°C)	Minimum strain $(\%)$	Mean strain $(\%)$	Maximum Strain (%)	Displacement (mm)	N_f (Cycles)
PVC	-40	3.8	6.3	9.0	2.0	990
		$1.4\,$	3.3	5.2	1.5	5,500
		$0.4\,$	1.6	$2.8\,$	1.5	27,000
	$25\,$	111.2	115.6	120.0	5.6	4,400
		77.0	82.5	88.0	5.6	32,000
		79.0	82.5	86.0	3.5	80,000
		$80.0\,$	82.5	85.0	$2.8\,$	300,000
	65	381.0	387.0	393.0	4.58	12,000
		384.0	387.0	390.0	3.08	450,000
		260.0	265.0	270.0	4.58	950,000
XLPE	-40	$10.0\,$	16.5	23.0	5.1	650
		9.3	13.2	17.1	$3.0\,$	7,600
		$8.8\,$	11.9	15.0	2.3	22,000
	$25\,$	216.0	220.0	224.0	5.1	25,000
		172.0	176.0	180.0	5.1	550,000
		52.0	56.0	60.0	5.1	1,010,000
	65	1160.0	1180.0	1200.0	5.1	6,500
		1020.0	1032.0	1044.0	3.1	198,000
		952.0	960.0	970.0	$2.0\,$	750,000
PPE	-40	6.2	12.6	19.0	5.1	40
		$1.2\,$	5.1	9.0	3.1	5,800
		0.7	3.6	6.5	2.28	25,000
	25	0.5	4.7	8.9	6.9	220
		0.1	2.3	4.5	3.3	8,200
		0.2	1.6	3.0	2.3	22,000
		$0.1\,$	$1.3\,$	$2.5\,$	1.9	70,000
	65	0.7	3.4	6.1	3.6	6,700
		$1.0\,$	2.4	3.8	$1.8\,$	42,000
		$\rm 0.8$	1.9	$3.0\,$	$1.5\,$	125,000

Table 2 Summary of different strain values applied on PVC during fatigue testing

specimen shows an improvement in fatigue life at both lower and higher temperatures compared to the room temperature. This appears to be an anomaly due to the brittle nature of PPE, as common in intrinsically brittle ceramics. There were a few instances in literatures [[16,](#page-7-0) [17\]](#page-7-0) where this behavior could be observed. It is found with a common trend that the fatigue life at a given temperature increases with a decrease of the maximum strain applied for all three materials. Studies on the effect of temperature on strain fatigue models await additional work and they are beyond the scope of this paper.

Modeling work

It is well known that the Coffin–Manson model is widely used to predict low-cycle fatigue life N_f of most metallic materials [\[18](#page-7-0), [19](#page-7-0)]. For low-cycle fatigue analysis, the strain-life behavior can be assumed by the total strain amplitude and cycles to failure as

$$
\frac{\Delta \varepsilon}{2} = C N_{\text{f}}^{-m} \tag{1}
$$

where $\Delta \varepsilon$ is the total strain, N_f the number of cycles to failure, m the fatigue exponent, and C the fatigue ductility coefficient. Clearly, the fatigue life N_f in Eq. 1 shows a linear relationship with the total strain amplitude $\Delta \epsilon/2$ in a log–log plot. The proportional relationship between the total strain amplitude and fatigue life had been proved for nylon materials in a log–log fashion [\[9](#page-7-0)].

Through a curve-fitting method, the relationships between $\Delta \varepsilon/2$ $\Delta \varepsilon/2$ and N_f are shown in Fig. 2 for PVC, Fig. [3](#page-3-0) for XLPE and Fig. [4](#page-3-0) for PPE in the log–log scale at three different temperatures, namely -40 , 25, and 65 °C, based on the experimental data of the three studied polymers. It is obvious that all trend lines are linear in the log–log plot.

Fig. 2 Total strain amplitude versus fatigue life for PVC at -40 , 25, and 65 °C. The trend lines are drawn by the least square fitting method. The trend line has a better linear correlation with the experimental data at low temperature, -40 °C

Fig. 3 Total strain amplitude versus fatigue life for XLPE at -40 , 25, and 65 °C. The constant strain amplitude at the room temperature has an infinite fatigue life, i.e., overestimation. Other trend lines are closer to the experimental data

Fig. 4 Total strain amplitude versus fatigue life for PPE at -40 , 25, and 65 °C. The trend lines display pronounced correlations with the experimental data

Cycles to Failure, Nf

Fig. 5 Mean/maximum strain versus fatigue life for PVC at -40 , 25, and 65 °C. The trend lines show better agreements with the experimental data than those in the strain amplitude model

Fig. 6 Mean/maximum strain versus fatigue life for XLPE at -40 , 25, and 65 °C. The horizontal trend line at the room temperature is replaced with a slope line for a finite fatigue life prediction

Using the same method, the maximum strain, ε_{max} , or mean strain, $\varepsilon_{\rm m}$, versus $N_{\rm f}$ are plotted in Fig. 5 for PVC, Fig. 6 for XLPE, and Fig. [7](#page-5-0) for PPE in the log–log scale at -40 , 25, and 65 °C, respectively. It is noted that there is a linear relationship between the logarithm of ε_{max} or ε_{m} and N_f for each trend line. Thus, taking ε_{max} as the vertical axis and N_f as the horizontal axis, the following expression can be made:

$$
\frac{d(\log(\varepsilon_{\max}))}{d(\log(N_f))} = -m \tag{2}
$$

where m is a constant for a given temperature. By integrating Eq. 2, fatigue life N_f can be obtained as

$$
\int_{\varepsilon_{\max 0}}^{\varepsilon_{\max}} d(\log(\varepsilon)) = -\int_{N_{\text{f}}}^{N_{\text{f}}} m d(\log(N_{\text{f}})) \tag{3}
$$

$$
\varepsilon_{\text{max}} = (\varepsilon_{\text{max}} \, {}_0N_{\text{f0}}^m)N_{\text{f}}^{-m} \tag{4}
$$

or

$$
\varepsilon_{\text{max}} = C \times N_f^{-m} \tag{5}
$$

where $C = \varepsilon_{\text{max}} \, {}_0N_{\text{f0}}^m$, $\varepsilon_{\text{max0}}$, and N_{f0}^m are the initial maximum strain and the number of cycles to failure, respectively. Through a least squares fitting, two material coefficients, C and m , for each of the studied conditions can be obtained as shown in Table [3.](#page-5-0) Similarly, a power-law relationship exists in ε_m and N_f , or ε_{\min} and N_f . Table [3](#page-5-0) also lists the material coefficients C and m for Coffin–Manson model, mean stain and minimum strain models.

It should be noted that only the effect from a singular source on the fatigue life is evaluated in all the equations discussed above. Accordingly, the total strain amplitude/ minimum strain model overestimates the fatigue life, especially for the constant amplitude case in strain amplitude model; whereas the maximum/mean strain model underestimates the fatigue life, especially for the influence of very small strain amplitudes [[20\]](#page-7-0). Neither of them Fig. 7 Mean/maximum strain versus fatigue life for PPE at -40 , 25, and 65 °C. It has better correlations with the experimental data than that in the strain amplitude model

Table 3 Material constants of C and m for PVC, XLPE, and PPE at each temperature

accurately predicts the fatigue lives with the constant/variable amplitude specimens. Therefore, a unified model, hereby named as the Wei–Wong model, integrating the effective strain effects, is proposed as follows:

$$
\left(\varepsilon_{\rm amp} \times \varepsilon_{\rm m} \times \varepsilon_{\rm max} \times \varepsilon_{\rm min}\right)^{\frac{1}{4}} = C N_{\rm f}^{-m} \tag{6}
$$

where $\varepsilon_{\rm amp}, \varepsilon_{\rm m}, \varepsilon_{\rm max}$, and $\varepsilon_{\rm min}$ represent the absolute values of strain amplitude, mean strain, maximum and minimum strains, respectively. Here we assume that $\varepsilon_m \neq 0$ and $\varepsilon_{\text{max}} \neq 0$ such as in tension–tension experiments. The unified strain model can be rendered consisting of three- or twostrain in the left-hand side of the equation, but the exponents will change to $1/3$ or $1/2$, accordingly, depending on the number of parameters used for the prediction. Note that the Wei–Wong model is not constrained to three or four independent parameters such as in the case as shown in Eq. 6. It is a multiparameter approach to addressing strain fatigue life of engineering polymers in contrast to the single-parameter concept in Eqs. [1](#page-2-0) and [5](#page-4-0). The merit of this unified model is it integrates the contributions from each of the parameters and thus minimizes the limitations arising from a singular source. In fact, the proposed model also provides a mean value for the exponent of the unified strains. Similarly, the two coefficients C and m in Table 3 at the selected temperatures can also be obtained through the least squares fits. For PVC, the results give the best fitting while including the strain amplitude, mean strain, and maximum strain. For XLPE and PPE, the results are best exemplified by the four-parameter equation. As discussed in Ref. [\[15](#page-7-0)], some tested specimens were pre-strained prior to fatigue testing to obtain the strain fatigue lives of the polymers or, otherwise, the specimens could not fail in a reasonable time frame for laboratory experiments. The ε_{max} after pre-straining is independent of both $\varepsilon_{\rm amp}$ and $\varepsilon_{\rm m}$. As a result, Eq. 6 herein consists of three independent parameters, namely $\varepsilon_{\rm{amp}}$, $\varepsilon_{\rm{m}}$, and $\varepsilon_{\rm{max}}$, and one dependent parameter ε_{\min} . This paper aims to illustrate the benefits of a multiparameter approach versus the singleparameter models existing in literature.

The Wei–Wong unified model for strain fatigue life prediction

As mentioned before, the total strain amplitude versus fatigue life in Fig. [2](#page-3-0) for PVC shows a decrease of total

strain amplitude as the number of cycles to failure increases. Note that it has a better linear correlation with the experimental data at low temperature, -40 °C. Although there are good agreements between the trend lines and experimental data for XLPE in Fig. [3,](#page-3-0) the constant strain amplitude at the room temperature causes an infinite fatigue life, i.e., overestimation. For PPE, the trend lines in Fig. [4](#page-3-0) display pronounced trends and have good agreements with the experimental data. It is noted that the strain amplitude model is inadequate to predict fatigue life for the constant amplitude situation.

The maximum and mean strain for PVC is plotted versus fatigue life at -40 , 2[5](#page-4-0), and 65 °C in Fig. 5. The solid trend line represents the maximum strain versus fatigue life. The dash trend lines are the mean strain versus fatigue life. The trend lines apparently have better agreements with the experimental data in the maximum and mean strain models besides at low temperature, compared with those in the strain amplitude model. For XLPE, note that the trend line in Fig. [6](#page-4-0) at room temperature exhibits a positive slope instead of the horizontal line for the constant amplitude, i.e., the fatigue life is finite to predict. The trend lines also fit well to the experimental data for PPE in Fig. [7.](#page-5-0) The maximum and mean strain models appear to successfully correlate well with the fatigue lives of the studied materials for the constant and varying strain amplitudes, and the strain curve.

fatigue life can be solved directly without the cyclic stress–
strain curve.
For comparisons, the calculated fatigue lives from
multistrain models are plotted versus the experimental lives
for PVC, XLPE, and PPE in Figs. For comparisons, the calculated fatigue lives from multistrain models are plotted versus the experimental lives for PVC, XLPE, and PPE in Figs. 8, 9, and [10](#page-7-0), respectively. In these figures, the perfect correlation is represented by data points lying on the diagonal line. The dashed line on each side of the diagonal represents error bands of 300, 300, and 40 cycles, respectively. In Fig. 8, the Wei–Wong unified strain model, involving the strain amplitude, mean, and maximum strains, shows the best fatigue life prediction for PVC, since their data points are in the error band. The single strain model has more data points outside the error band than the unified strain model. Figure 9 shows three infinite points for the constant amplitude in strain amplitude model; whereas other strain models have no infinite points. The strain models, including the unified strain model, have almost the same predicted lives for XLPE, except for the strain amplitude model due to the effect of the constant amplitude. Note that the Wei–Wong unified model inclusive of four strain parameters shows the best correlation with the experimental data for PPE in Fig [10](#page-7-0). From these figures, fatigue lives obtained from the unified strain model exhibits a better correlation with the experimental lives than a singleparameter model. It is noteworthy that the unified model is based on a mean value concept of the exponents and

Fig. 8 Comparison of predicted lives by different strain models with experimental lives for PVC at -40 , 25, and 65 °C. The unified strain model shows the best fatigue life prediction as all of star points fall in the error band

Fig. 9 Comparison of predicted lives by different strain models with experimental lives for XLPE at -40 , 25, and 65 °C. The strain amplitude model has three infinite points for the constant amplitude; whereas other strain models have no infinite points. The strain models except for the strain amplitude model have almost the same predicted lives

logarithms for both material constants in Eq. [6.](#page-5-0) For example, the fatigue exponent ($m = 0.335$) for PVC at -40 °C in the unified model (including strain amplitude, mean strain and maximum strain) in Table [3](#page-5-0) equals the

Fig. 10 Comparison of predicted lives by different strain models with experimental lives for PPE at -40 , 25, and 65 °C. The unified model including four strains has the best correlations with the experimental fatigue life among strain models

mean value of exponents in three of the single strain models. The ductility coefficient $(C = 54.69)$ equals the mean value of logarithm of the three single-parameter models.

It is also noted that the more strains we unify, the more accurate we can predict the strain fatigue life, as evident in Figs. [8](#page-6-0) and 10. Most of predicted data points are located in the error band because of the averaging effect of unified model. However, the unified strain model has almost the same predicted data with the single strain model except for the strain amplitude model in XLPE in Fig. [9,](#page-6-0) because of the infinite life effect from the constant amplitude. Therefore, the Wei–Wong unified strain model integrates merits of the single strain models, and minimizes their limitations.

Conclusions

In this study, low-cycle fatigue tests and modeling on three insulation plastics, namely PVC, XLPE, and PPE, were carried out at -40 , 25, and 65 °C. All strain models, including total strain amplitude, maximum strain, minimum strain, and mean strain models, were analyzed and compared with the experimental data.

Although the Coffin–Manson model, considering total strain amplitude influence, exhibits a good agreement with experimental data on fatigue life in some cases such as in low temperature -40 °C and under the variable amplitude loading, it appears inaccurate in predicting our experimental data [15], especially for the constant amplitude case. The

maximum/mean strain models showed better predictions than the single strain amplitude model under constant and variable amplitude loading.

The Wei–Wong unified strain model was proposed to integrate the contributions from two or higher order strain fatigue parameters to take into account of the multiple strain effects on fatigue life. It is used with the assumptions of nonzero mean and minimum strains. It was noted from this study that the fatigue lives predicted by the unified strain model are in a much closer agreement with the experimental results than the single-parameter models for PVC, XLPE, and PPE.

It is also observed that the more strains we unify, the more accurate the model can predict the strain fatigue lives of engineering polymers. The semi-empirical unified model integrates the merits from each single strain contribution, and minimizes their drawbacks. It presents a simple and useful method for fatigue life prediction. However, additional studies especially at high temperature are required to warrant their applicability to insulation materials outside the scope of our present investigation.

Acknowledgments The authors wish to thank Pei Chen and Paul Lam for assisting in the preparation of this manuscript.

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